**MAA507 – SPRING 2022**

**SEMINAR 3 PREPARATION EXERCISES**

**1.1**

Betweenness of the edges:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Initial | After removal of CD | After removing AC, BC |
| AC | 8 | 2 |  |
| BC | 8 | 2 |  |
| CD | 16 |  |  |
| DE | 6 | 0 | 0 |
| DF | 6 | 0 | 0 |
| EF | 0 | 0 | 0 |
| Edge(s) to be removed | CD | AC, BC | DE, DF, EF |
| Graph |  |  |  |

To make the algorithm deterministic, I preferred to remove all the edges in case of equality.

**1.2**

As explained in the lecture 8 slides:

Steps:

1. Add self-loops to every vertex
2. Convert the graph to an adjacency matrix such that values in each row add up 1.
3. Expand the matrix to the power of k
4. Take each element of the matrix to the power of alpha
5. Make the sum of each row 1
6. Repeat 3-4-5

Iteration of the algorithm on the graph:

Create the adjacency matrix

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | A | B | C | D | E | F |
| A | 1 | 0 | 1 | 0 | 0 | 0 |
| B | 0 | 1 | 1 | 0 | 0 | 0 |
| C | 1 | 1 | 1 | 1 | 0 | 0 |
| D | 0 | 0 | 1 | 1 | 1 | 1 |
| E | 0 | 0 | 0 | 1 | 1 | 1 |
| F | 0 | 0 | 0 | 1 | 1 | 1 |

Rescale the matrix so that the sum of each row is 1

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | A | B | C | D | E | F |
| A | 0.5 | 0 | 0.5 | 0 | 0 | 0 |
| B | 0 | 0.5 | 0.5 | 0 | 0 | 0 |
| C | 0.25 | 0.25 | 0.25 | 0.25 | 0 | 0 |
| D | 0 | 0 | 0.25 | 0.25 | 0.25 | 0.25 |
| E | 0 | 0 | 0 | 0.33 | 0.33 | 0.33 |
| F | 0 | 0 | 0 | 0.33 | 0.33 | 0.33 |

Expansion by k = 2 (Namely squaring)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | A | B | C | D | E | F |
| A | 0.375 | 0.125 | 0.375 | 0.125 | 0 | 0 |
| B | 0.125 | 0.375 | 0.375 | 0.125 | 0 | 0 |
| C | 0.1875 | 0.1875 | 0.375 | 0.125 | 0.0625 | 0.0625 |
| D | 0.0625 | 0.0625 | 0.125 | 0.29 | 0.2275 | 0.2275 |
| E | 0 | 0 | 0.0825 | 0.3003 | 0.3003 | 0.3003 |
| F | 0 | 0 | 0.0825 | 0.3003 | 0.3003 | 0.3003 |

Inflation by alpha = 2

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | A | B | C | D | E | F |
| A | 0.140 | 0.015 | 0.140 | 0.015 | 0 | 0 |
| B | 0.015 | 0.140 | 0.140 | 0.015 | 0 | 0 |
| C | 0.035 | 0.035 | 0.140 | 0.015 | 0.003 | 0.003 |
| D | 0.003 | 0.003 | 0.015 | 0.084 | 0.051 | 0.051 |
| E | 0 | 0 | 0.006 | 0.090 | 0.090 | 0.090 |
| F | 0 | 0 | 0.006 | 0.090 | 0.090 | 0.090 |

Rescale the rows

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | A | B | C | D | E | F |
| A | 0.448 | 0.048 | 0.448 | 0.048 | 0 | 0 |
| B | 0.048 | 0.448 | 0.448 | 0.048 | 0 | 0 |
| C | 0.151 | 0.151 | 0.304 | 0.064 | 0.012 | 0.012 |
| D | 0.014 | 0.014 | 0.072 | 0.405 | 0.246 | 0.246 |
| E | 0 | 0 | 0.021 | 0.325 | 0.325 | 0.325 |
| F | 0 | 0 | 0.021 | 0.325 | 0.325 | 0.325 |

**1.3**

Linkage criterion takes an important role in clustering because it may completely change the distribution of the clusters. Complete linkage, in which the maximum distance is considered, leads to circular-shaped clusters. On the other hand, if single linkage is used, the minimum distance will be considered, and this causes the clusters to have irregular shapes. Because the cluster boundary will be more changeable with new merges. Mean linkage will be in between complete and single linkages.